SIGNIFICANT FIGURES, EXPONENTS, AND SCIENTIFIC NOTATION

1. SIGNIFICANT FIGURES

Significant figures are those digits necessary to express the results of a measurement to the precision with which it was made. No measurement is ever absolutely correct since every measurement is limited by the accuracy or reliability of the measuring instrument used. For example, if a thermometer is graduated in one degree intervals and the temperature indicated by the mercury column is between 55°C and 56°C, then the temperature can be read precisely only to the nearest degree (55°C or 56°C, whichever is closer). If the graduations are sufficiently spaced, the fractional degrees between 55°C and 56°C can be estimated to the nearest tenth of a degree. If a more precise measurement is required, then a more precise measuring instrument (e.g., a thermometer graduated in one-tenth degree intervals) can be used. This will increase the number of significant figures in the reported measurement. (See Figure 1)

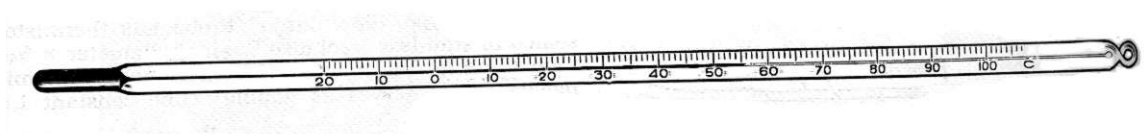

Figure 1. A typical Laboratory Thermometer graduated in °C.

In dealing with measurements and significant figures the following terms must be understood:

> **Precision** tells the reproducibility of a particular measurement or how often a particular measurement will repeat itself in a series of measurements.
>
> **Accuracy** tells how close the measured value is to a known or standard accepted value of the same measurement.

Measurements showing a high degree of precision do not always reflect a high degree of accuracy nor does a high degree of accuracy mean that a high degree of precision has been obtained. It is quite possible for a single, random measurement to be very accurate as well as to have a series of highly precise measurements be inaccurate. Ideally, high degrees of accuracy and precision are desirable, and they usually occur together, but they are not always obtainable in scientific measurements.

Every measuring device has a series of markings or **graduations** on it that are used in making a measurement. The precision of any measurement depends on the size of the graduations. The smaller the interval represented by the graduation, the more precise the possible measurement.

However, depending on the size of the graduations, and, as a general rule, *any* measurement can only be precise to ± ½ of the smallest graduation on the measuring device used, provided that the graduations are sufficiently close together. In the cases where the measurement intervals represented by the graduations are sufficiently large, you may be able to estimate the tenths of the graduation, then the uncertainty of that measurement can be considered to be ± one unit of the last digit in the recorded measurement. (See Figures 2 and 3)

The accuracy of a measuring device depends on how exact the graduations are marked or engraved on the device in reference to some standard measurements. For most measuring devices used in everyday work, the graduations on them are usually sufficiently accurate for general use. In the laboratory, it is not always advisable to accept a measuring device as accurate unless the instrument has been calibrated. **Calibration** is the process of checking the graduations on a measuring device for accuracy. As an example, consider a thermometer which is graduated in

Celsius degrees. When this thermometer is placed in an ice bath at 0°C, it reads -1°C and when placed in boiling water at 100°C, it reads 99°C. This thermometer has been roughly calibrated over a 100° temperature range and has been found to be 1° in error. As a correction factor, 1° must be added to all temperature readings in this temperature range. It would be better, however, to check the thermometer at several different temperatures within this range to verify that the error is indeed linear before the uniform application of the correction factor at all temperatures.

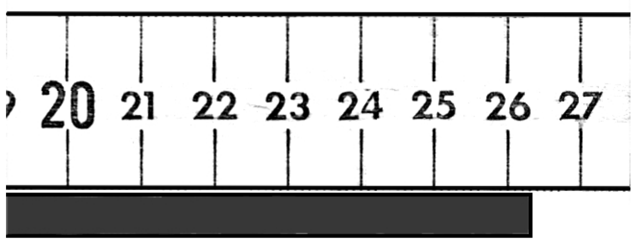

Figure 2 A meter stick graduated in centimeters. The graduations are sufficiently far apart that it is relatively easy to estimate the length of the object shown above it. The length of the object is estimated to be 26.3±0.1 cm. The doubtful or uncertain digit here is the last one recorded.

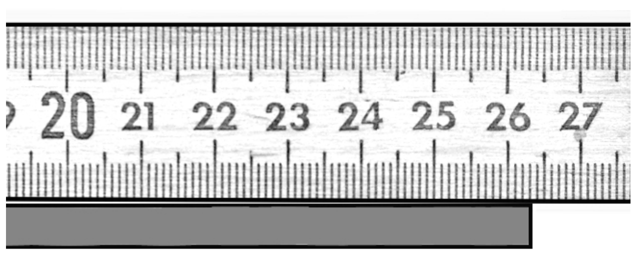

Figure 3 A meter stick graduated in millimeters. The graduations are close enough together that it is difficult to estimate to the nearest tenth of a millimeter. In reporting the length of the object shown above the meter stick, it is sufficient to report it to the nearest millimeter. Thus the length is reported to be 26.3±0.1 cm. The uncertain digit here is the smallest -graduation on the meter stick, ±0.1 cm (or ±1 mm).

When using a meter stick, even one that is finely machined, there is enough unevenness in the object being measured, the meter stick itself, and the difficulty in placing it on the item to be measured to make it pointless to attempt to estimate tenths of a millimeter. Even under ideal conditions, estimation of anything less than one-half a millimeter becomes more of a guess than a measurement.

Whether the information from a series of measurements is obtained first-hand or second hand through another source, the number of significant figures must be determined in order to keep all the results meaningful. The rules for writing and identifying significant figures are:

1. All nonzero digits (digits from 1 to 9) are significant.

   254 contains **three** significant figures
   4.55 contains **three** significant figures
   129.454 contains **six** significant figures

2. Zero digits that occur between nonzero digits are significant.

202 contains **three** significant figures 450.5 contains **four** significant figures 390.002 contains **six** significant figures	In these examples, the zeros are part of a measurement.

3. Zeros at the beginning of a number (i.e., on the left-hand side) are considered to be placeholders and are not significant.

0.00078 contains **two** significant figures 0.00205 contains **three** significant figures 0.0302 contains **three** significant figures	In these examples, the zeros on the left are placeholders.

It is common practice to place a zero in front of the decimal point preceding a decimal fraction. It acts as a placeholder only.

4. Zeros that occur at the end of a number (i.e., on the right-hand side) that include an expressed decimal point are significant. The presence of the decimal point is taken as an indication that the measurement is exact to the places indicated.

57500. contains **five** significant figures 2000. contains **four** significant figures 34.00 contains **four** significant figures 25.200 contains **five** significant figures. 0.002050 contains **four** significant figures	In these examples, the zeros on the right express part of a measurement.

5. Zeros that occur at the end of a number (i.e., on the right-hand side) without an expressed decimal point are ambiguous (*i.e.*, we have no information on wether they are significant or not) and are **not** considered to be significant.

575000 contains three significant figures 2000 contains one significant figure 40620 contains four significant figures	In these examples, the zeros may only be placeholders. Do not count them unless a decimal point is present.

One way of indicating that some or all the zeros are significant is to write the number in scientific notation form (See Section 6). For example, a number such as 2000 would be written as 2×10^3 if it contains one significant figure, and as 2.0×10^3 if it contains two significant figures.

Problems: Significant Figures

State the number of significant figures in each of the following measurements.

a) 230 cm ans. a)____________________

b) 34.0 mL b)____________________

c) 0.625 g c)____________________

d) 56.0030 g d)____________________

e) 83400 km e)____________________

f) 4200. mL f)____________________

g) 0.000 620 m g)___________________

h) 0.0004 g h)___________________

2. ROUNDING-OFF NUMBERS

When dealing with significant figures, it is often necessary to round-off numbers in order to keep the results of calculations significant. To round-off a number such as 64.82 to three significant figures means to express it as the nearest three digit number. Since 64.82 is between 64.8 and 64.9, but closer to 64.8, then the result of the round-off is 64.8.

A number such as 64.85 is equally close to 64.8 and 64.9. In this case and in similar cases, the rule to observe is to round off to the **nearest even number** which is 64.8. This rule assumes that in a series of numbers which are to be rounded off, there will be approximately the same number of times that you would have to round-off upward to the nearest even number as you would have to round-off downward.

Examples:

1. Round off 75.52 to three significant figures.

   Answer: 75.52 is between 75.5 and 75.6. Since 75.52 is closer to 75.5, then the answer is <u>75.5</u>

2. Round off 9.08352 to two decimal places.

   Answer: When expressed to two decimal places, 9.08352 falls between 9.08 and 9.09. Since it is closer to 9.08, then the answer is <u>9.08</u>

3. Round off 1345.54 to a whole number.

   Answer: 1345.54 is between 1345 and 1346. The number, 1345.54 is closer to 1346, thus, the answer is <u>1346</u>

4. Round off 7962400 to three significant figures.

   Answer: To round off 7962400, use the first three significant figures. Therefore, 7962400 is between 7960000 and 7970000, but it is closer to 7960000. The zeros must be maintained as placeholders. The answer is <u>7960000</u>

5. Round off 0.000275 to two significant figures.

   Answer: 0.000275 is between 0.00027 and 0.00028. Since it is equally close to both these numbers, then it will be rounded off to the nearest *even* number. Round off upward to <u>0.00028</u>

Problems: Rounding-off numbers

Round off each of the following numbers to three significant figures.

a) 63.351 ans. a)_________________

b) 0.0000004399 b)_________________

c) 10249000 c)_________________

d) 555.50 d)__________________

e) 0.0020285 e)__________________

f) 90960 f)__________________

g) 3.79745 g)__________________

h) 7296.38 h)__________________

3. NUMERICAL OPERATIONS WITH SIGNIFICANT FIGURES

A. Addition and subtraction

When adding (or subtracting) approximate numbers, round off the sum (or difference) to the last column in which each number has a significant figure.

Examples:

1. Add the following numbers: 67.25 + 721.2 + 16530.006 + 282.43

Answer:

First arrange the numbers in a column:

```
                    67.25
                   721.2
                 16530.006
                   282.43
The sum is       17600.886
```

Applying the rule for addition of significant figures, it is observed that the last column in which every one of the four numbers has a significant figure is the tenths column (the first decimal place). Thus the sum must be rounded off to one decimal place.

The answer would properly be reported as 17600.9 (*6 significant figures*)

2. Subtract the following numbers: 978.4 - 62.87

Answer:

Set up the subtraction problem:

```
                     978.4
                   - 62.87
The difference is    915.53
```

The last column in which both of these numbers contain a significant figure is the tenths column. Round off the answer to one decimal place.

The difference is properly reported as 915.5 (*4 significant figures*)

B. Multiplication and division

When multiplying (or dividing) approximate numbers, round off the product (or -quotient) so that the final answer contains only as many significant figures as the least approximate number involved in the calculation.

Examples:

1. Multiply the following numbers: 3.01 x 1.4 x 725.1

   Answer:

   The product of multiplication is:

   3.01 x 1.4 x 725.1 = 3055.5714

   The process of multiplication can result in many more digits in the answer than in any of the original numbers in the problem. If we examine each of the initial numbers in the problem, we find 3.01 contains 3 significant figures; 1.4 contains 2 significant figures; and 725.1 contains 4 significant figures. The number of significant figures in the answer will be determined by the least approximate number, the 1.4 which contains 2 significant figures.

   The final product is rounded off and reported as <u>3100</u> (*2 significant figures*)

2. Divide the following numbers: 3.1416 / 6.01

   Answer:

   The quotient of this division problem is:

   3.1416 / 6.01 = 0.52273

   Examining the initial numbers, you will observe that 3.1416 contains 5 significant figures and 6.01 contains 3 significant figures. Thus, 6.01 is the least approximate number and the quotient is rounded off to 3 significant figures.

   The final quotient will be reported as <u>0.523</u> (*3 significant figures*)

Problems: Numerical operations with significant figures

Perform the indicated mathematical operations in each of the following. Round off the answers to the proper number of significant figures.

a) 501.2 g + 32.346 g + 12.33 g

b) 14.25 cm - 2.234 cm

c) 75.5 m x 8.66 m x 44 m

d) 96.435 g/ 3.45 g

e) 2334 cm x 1.020 cm x 21.2 cm

f) 8.6 mL + 0.3520 mL + 70.55 mL

g) 0.00164 L/ 0.0004 L

h) 9.450 cm - 0.3 mm

4. EXPONENTS

Occasionally, in scientific work, we encounter a product in which the same number is used more than once as a factor, as in the following two examples:

$$2 \times 2 \times 2 \times 2 \qquad \text{or} \qquad 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

This method of writing such numbers is cumbersome and in order to simplify this, we use exponential notation. The two examples given above can be written in exponential notation as:

2^4 where the "4" means that there are four "2's" to be multiplied together.

3^6 where the "6" means that six "3's" are to be multiplied together.

The answers to these examples are:

$$2^4 = \underline{16} \qquad \text{and} \qquad 3^6 = \underline{729}$$

In general form an exponential number is given as:

$$a^n = a \times a \times a \times a \times a \ldots \times a \quad \text{(n times)}$$

where the symbol a^n is read as the "n^{th} *power of* a" or "a *to the* n^{th} ".

The repeated factor, **a**, is called the **base** and the number of factors to be multiplied together, **n**, is called the **exponent**.

Two exponents have common names:

a^2 is usually read as "a *squared*"
and
a^3 is usually read as "a *cubed*"

The types of exponents that you will be encountering in scientific work will be described in the following sections. In most cases, we will be mainly concerned with exponential powers of ten.

A. Positive Exponents

If the exponent, n, is a positive integer, then a^n denotes the product of n factors where each factor is equal to a. This can be written in the general form:

$$a^n = a \times a \times a \times a \ldots \times a \text{ (n times)}$$

Examples:

4^3 can be written as the product of *three* fours:

$$4^3 = 4 \times 4 \times 4 = \underline{64}$$

10^3 can be written as the product of *three* tens:

$$10^3 = 10 \times 10 \times 10 = \underline{1000}$$

10^6 can be written as the product of *six* tens:

$$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = \underline{1\ 000\ 000}$$

A summary of some positive powers of ten are given below:

10^1	=	10
10^2	=	100
10^3	=	1 000
10^4	=	10 000
10^5	=	100 000
10^6	=	1 000 000
10^7	=	10 000 000
10^8	=	100 000 000
10^9	=	1 000 000 000

Examining the numbers above, you may observe that there is a relationship between the positive exponent of ten and the number of zeros in the product. This can be summarized as:

> *For powers of ten where the exponent is a positive integer,* **n**, *the product can be written as a 1 with* **n** *zeros following it.*

Occasionally, one encounters fractions that are raised to a power. An example of this is:

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} \quad \frac{8}{27}$$

B. Zero Exponents

If the exponent, n, is zero, then by definition, any number raised to the zero power is equal to unity (*i.e.*, 1).

Examples:

$$10^0 = 1$$
$$6^0 = 1$$
$$143.25^0 = 1$$
$$(4 \times 10^3)^0 = 1$$

C. Negative Exponents

If the exponent, n, is a negative quantity, then the expression a^{-n} can be written as shown below:

$$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$$

Note that when the number with a negative exponent is rewritten in fractional form, the sign of the exponent becomes positive.

Examples:

3^{-2} can be evaluated as shown below:

$$3^{-2} = \frac{1}{3} = \frac{1}{3 \times 3} = \frac{1}{9}$$

10^{-1} can be rewritten and evaluated as shown below:

$$10^{-1} = \frac{1}{10} = \underline{0.1}$$

10^{-3} can be written and evaluated as shown below:

$$10^{-3} = \frac{1}{10^3} = \frac{1}{10 \times 10 \times 10} = \frac{1}{1000} = \underline{0.001}$$

A summary of some negative powers of ten are given below:

10^{-1} = 0.1
10^{-2} = 0.01
10^{-3} = 0.001
10^{-4} = 0.000 1
10^{-5} = 0.000 01
10^{-6} = 0.000 001
10^{-7} = 0.000 000 1
10^{-8} = 0.000 000 01
10^{-9} = 0.000 000 001

Examining the numbers above, you may observe that there is a relationship between the negative exponent of ten and the number of zeros in the product. This can be summarized as:

> *For powers of ten where the exponent is a negative integer,* **-n**, *the product can be written as a decimal fraction with* **(n-1)** *zeros between the decimal point and the* **1**.

Occasionally, one encounters fractions that are raised to a negative power. An example of this is:

$$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{3 \times 3 \times 3}{2 \times 2 \times 2} = \frac{27}{8}$$

D. Fractional Exponents

If the exponent is a fraction of the form $\frac{m}{n}$, where m and n are integers, then an

exponential number such as $a^{m/n}$ is read as "*take the n^{th} root of* **a** *and raise it to the m^{th} power*". This can be written in the form:

$$a^{m/n} = (\sqrt[n]{a})^m$$

Examples:

$10^{1/2}$ can be rewritten and evaluated as shown below:

$$10^{1/2} = \sqrt{10} = \underline{3.16}$$

$8^{2/3}$ is evaluated as shown below:

1/4

$$8^{2/3} = (\sqrt[3]{8})^2 = (2)^2 = \underline{4}$$

$\left(\frac{1}{81}\right)^{1/4}$ is evaluated:

$$\left(\frac{1}{81}\right)^{1/4} = \sqrt[4]{\frac{1}{81}} = \frac{\sqrt[4]{1}}{\sqrt[4]{81}} = \frac{1}{3}$$

Problems: Exponents

Evaluate the following exponential numbers:

a) 3^4

b) 5^3

c) 10^2

d) 10^5

e) 10^8

f) $\left(\frac{3}{4}\right)^2$

g) 3^{-2}

h) 2^{-5}

i) 10^{-2}

j) 10^{-5}

k) 10^{-8}

l) $\left(\frac{3}{4}\right)^{-2}$

m) $64^{2/3}$

o) $4^{-3/2}$

5. MATHEMATICAL OPERATIONS WITH EXPONENTIAL NUMBERS

This tutorial will address multiplication, division, and powers with exponential numbers. Addition and subtraction with exponential numbers is not commonly encountered in chemistry courses.

A. Multiplication

To multiply two exponential numbers having the same base, add the exponents:

$$a^n \times a^m = a^{(n+m)}$$

Examples:

Evaluate $3^2 \times 3^3$

$$3^2 \times 3^3 = 3^{(2+3)} = 3^5 = \underline{243}$$

Evaluate $3^2 \times 3^{-3}$

$$3^2 \times 3^{-3} = 3^{2+(-3)} = 3^{-1} = \frac{1}{3}$$

Note: If one or more of the exponents have a negative sign, addition of exponents is with respect to sign.

Evaluate $10^2 \times 10^3 \times 10^4$

$$10^2 \times 10^3 \times 10^4 = 10^{2+3+4} = 10^9 = \underline{1\ 000\ 000\ 000}$$

Evaluate $10^2 \times 10^{-3} \times 10^{-4}$

$$10^2 \times 10^{-3} \times 10^{-4} = 10^{2+(-3)+(-4)} = 10^{-5} = \underline{0.000\ 01}$$

B. Division

To find the quotient of two exponential numbers having the same base, subtract the exponent of the denominator form the exponent of the numerator:

$$\frac{a^n}{a^m} = a^{n-m}$$

Examples:

Evaluate $5^6/5^3$

$$5^6/5^3 = \frac{5^6}{5^3} = 5^{6-3} = 5^3 = \underline{125}$$

Evaluate $5^3/5^{-2}$

$$5^3/5^{-2} = \frac{5^3}{5^{-2}} = 5^{3-(-2)} = 5^{3+2} = 5^5 = \underline{3125}$$

Evaluate $10^8/10^{12}$

$$10^8/10^{12} = \frac{10^8}{10^{12}} = 10^{8-12} = 10^{-4} = \underline{0.0001}$$

C. Raising to Powers

To raise a exponential number to a power, multiply the exponents:

$$(a^n)^m = a^{n \cdot m}$$

Examples:

Evaluate $(2^3)^2$

$$(2^3)^2 = 2^{3 \cdot 2} = 2^6 = \underline{64}$$

Evaluate $(10^4)^3$

$$(10^4)^3 = 10^{4 \cdot 3} = 10^{12} = \underline{1\ 000\ 000\ 000\ 000}$$

Evaluate $(10^{-3})^2$

$$(10^{-3})^2 = 10^{(-3) \cdot 2} = 10^{-6} = \underline{0.000\ 001}$$

Evaluate $(10^{1/2})^8$

$$(10^{1/2})^8 = 10^{(1/2) \cdot 8} = 10^4 = \underline{10\ 000}$$

Problems: Mathematical operations with exponential numbers

Evaluate each of the following:

a) $2^3 \times 2^2$

b) $4^5 \times 4^{-3}$

c) $2^{-5}/ 2^{-3}$

d) $4^3/4^5$

e) $(2^{-2})^2$

f) $(4^2)^5$

g) $10^2 \times 10^6$

h) $10^3 \times 10^5 \times 10^{-4}$

i) $10^{-3} \times 10^{-2}$

j) $10^4/10^7$

h) $10^2/10^{-5}$

l) $10^{-4}/10^{-7}$

m) $(10^3)^2$

n) $(10^3)^{-4}$

o) $(10^{-3})^{-2}$

p) $\dfrac{(10^{-2} \times 10^6)}{10^8}$

6. SCIENTIFIC NOTATION

In chemistry, we frequently have to deal with very large or very small numbers. For example, one mole of any substance contains approximately

602 000 000 000 000 000 000 000 particles

of that substance. If we consider a substance such as gold, one atom of gold will weigh

0.000 000 000 000 000 000 000 327 grams

Numbers such as these are difficult to write and are even more difficult to work with, especially in calculations where the number may have to be used a few times. To simplify working with these numbers, we use what is known as **scientific notation**.

In scientific notation we make the assumption that a number such as

25 000 000

can be written as the product of two numbers

2.5 and 10 000 000

To further simplify this expression, the number 10 000 000 can be written in exponential form. Thus, this number, in scientific notation becomes

$$25\ 000\ 000 = 2.5 \times 10^{7}$$

In proper scientific notation form, the significant figures are written so that the decimal point is located **between the first and second significant figures** (counting from left to right). The power of ten is the indicator of how many spaces the decimal point had to be moved to place it between the first two significant figures.

In the first number, the number of particles in a mole, given above, in order to place the decimal point between the first two significant figures (the 6 and the 0) the decimal point must be moved twenty-three spaces form the right to the left. The number of spaces the decimal point is moved will be expressed as a positive integer. In proper scientific notation form, the number of particles in a mole is

6.02×10^{23} particles

In the second number, the number of grams in one atom of gold, the decimal point must be moved twenty-two spaces form the left to the right. The number of spaces the decimal point was moved will be expressed as a negative exponent. The weight of one atom of gold, expressed in scientific notation.

3.27×10^{-22} gram

As you can observe, these two numbers are easier to manage in scientific notation.

Some more examples of numbers written in scientific notation are:

$4500 = 4.5 \times 10^{3}$ (decimal point moved 3 spaces to the left)

$305\ 000 = 3.05 \times 10^{5}$ (decimal point moved 5 spaces to the left)

$0.00250 = 2.50 \times 10^{-3}$ (decimal point moved 3 spaces to the right)

Remember, if the *decimal point was moved from the right to the left* in order to place it between the first two significant figures, *the exponent will be positive*. If the *decimal point had to be moved from the left to the right, the exponent will be negative.*

Another advantage of scientific notation is that *only the significant figures are kept*, all placeholders are contained in the power of ten. This eliminates zeros which are ambiguous. For example, if the number 23985 is rounded off to three significant figures, the result would be 24000. Only the zero following the 4 should be significant, the other zeros are place holders. It is not apparent from looking at the number that one of the zeros is significant while the others are not. In scientific notation, this number would be written as

$$2.40 \times 10^4$$

Thus, the significant zero is included, but the placeholders are not.

Problems: Scientific notation

1. Write the following numbers in proper scientific notation form:

a) 662 500 000 000 — ans. a)__________________

b) 0.000 000 035 60 — b)__________________

c) 0.025 — c)__________________

d) 9800 — d)__________________

e) 2025×10^3 — e)__________________

f) 0.0980×10^{-2} — f)__________________

2. Round off the following numbers as indicated and write the answer in scientific notation form:

a) Round off 45 379 662 to 3 significant figures — ans. a)__________________

b) Round off 739966 to 4 significant figures — b)__________________

c) Round off 0.025988 to 3 significant figures — c)__________________

d) Round off 0.000 098 726 to 3 significant figures — d)__________________

3. Express the following exponential numbers in non-exponential form:

a) 7.90×10^5 — ans. a)__________________

b) 5.70×10^{-4} — b)__________________

c) 4.550×10^{-9} — c)__________________

d) 3.000×10^8 — d)__________________

e) 9.09×10^{-3} — e)__________________

7. NUMERICAL OPERATIONS WITH SCIENTIFIC NOTATION NUMBERS

A. Multiplication

To multiply two (or more) numbers written in scientific notation form, first rearrange the numbers grouping the non-exponential parts of the numbers together and grouping the powers of ten together. The non-exponential numbers are multiplied together and the exponential numbers are added.

Examples:

Multiply: $(3 \times 10^{12}) \times (2 \times 10^{6})$

Answer:
$$\begin{aligned}(3 \times 10^{12}) \times (2 \times 10^{6}) &= 3 \times 10^{12} \times 2 \times 10^{6} \\ &= 3 \times 2 \times 10^{12} \times 10^{6} \quad \text{(rearrange)} \\ &= 6 \times 10^{12+6} \quad \text{(multiply)} \\ &= \underline{6 \times 10^{18}}\end{aligned}$$

Multiply: $(4.50 \times 10^{-8}) \times (3.00 \times 10^{4})$

Answer:
$$\begin{aligned}(4.50 \times 10^{-8}) \times (3.00 \times 10^{4}) &= 4.50 \times 10^{-8} \times 3.00 \times 10^{4} \\ &= 4.50 \times 3.00 \times 10^{-8} \times 10^{4} \\ &= 13.5 \times 10^{-8+4} \\ &= 13.5 \times 10^{-4} \\ &= \underline{1.35 \times 10^{-3}}\end{aligned}$$

Do not forget to put the final answer in proper scientific notation form (i.e., one number in front of the decimal place.)

B. Division

To divide two numbers written in scientific notation, first rearrange them to group the non-exponential numbers together and the powers of ten together. The non-exponential numbers are divided and the powers of ten are subtracted.

Examples:

Divide: $(3.0 \times 10^{12}) / (2.0 \times 10^{16})$

Answer:
$$\begin{aligned}\frac{3.0 \times 10^{12}}{2.0 \times 10^{16}} &= \frac{3.0}{2.0} \times \frac{10^{12}}{10^{16}} \quad \text{(rearrange)} \\ &= 1.5 \times 10^{12-16} \quad \text{(divide)} \\ &= \underline{1.5 \times 10^{-4}}\end{aligned}$$

Divide: $(6.60 \times 10^{-3}) / (2.20 \times 10^{4})$

Answer: $$\frac{6.60 \times 10^{-3}}{2.20 \times 10^{4}} = \frac{6.60}{2.20} \times \frac{10^{-3}}{10^{4}}$$

$$= 3.00 \times 10^{-3-(4)}$$

$$= \underline{3.00 \times 10^{-7}}$$

C. Raising to a Power

To raise a number in scientific notation to a power, separate the non-exponential part from the power of ten, raise the non-exponetial part to the required power, and multiply the exponents.

Example:

Evaluate $(3.0 \times 10^{2})^{3}$

Answer: $(3.0 \times 10^{2})^{3}$ $= (3.0)^{3} \times (10^{2})^{3}$
$= 27 \times 10^{2 \cdot 3}$
$= 27 \times 10^{6}$
$= \underline{2.7 \times 10^{7}}$

Problems: Numerical operations with scientific notation numbers

Evaluate each of the following and express the answers in proper scientific notation form.

a) $(4.50 \times 10^{6}) \times (2.1 \times 10^{-2})$

b) $(5.50 \times 10^{-3}) \times (3.50 \times 10^{-4})$

c) $(1.5 \times 10^{3}) \times (6.6 \times 10^{4})$

d) $9000 / (2.5 \times 10^{2})$

e) $(4.48 \times 10^{-3}) / (6.60 \times 10^{2})$

f) $(8.5 \times 10^{-5}) / (3.5 \times 10^{-8})$

g) $(2.00 \times 10^{-3})^{4}$

h) $(3.0 \times 10^{4})^{3}$

i) $(1.60 \times 10^{-5})^{-3}$

j) $\dfrac{136\ 000 \times 0.00030 \times 150}{0.080 \times 4200 \times 75000}$

k) $(8\ 000\ 000)^{2/3}$

l) $\dfrac{(0.000\ 000\ 4)^3 \times (6000)^2}{(0.000\ 02)^4 \times (400)^{1/2}}$

m) $(\sqrt[3]{8\ 000\ 000\ 000}) \times (\sqrt[3]{0.027})$

n) $4000 / (2.12 \times 10^{-3})^0$