PROBLEM SOLVING BY DIMENSIONAL ANALYSIS

Problem solving in chemistry almost always involves word problems or "story-problems". Although there is no single method for solving all types of problems encountered in this course, the method known as *dimensional analysis* or the *unit-factor* method involves problem solving techniques that can be applied to many different types of problems. To illustrate this type of problem solving, problems involving metric conversions will be used.

Often, it is necessary to convert measurements expressed in one unit (such as mm) to another unit (such as cm, m, or km). Such conversions are carried out using **conversion factors** which are derived from or given in tables in the section on the SI system. The method of dimensional analysis involves working with these conversion factors and canceling physical units that accompany the numbers or measurements along with the numbers themselves.

To illustrate the conversion process, we will start with a simple conversion problem:

Example 1:
A bar of magnesium metal is determined to be 250. mm long. What is the length of the magnesium bar in m?

The steps in solving the problem are:

1. What is the question?

The first step in solving a problem is to identify the problem or question. Look for key words or phrases such as *convert, change, express, what is, or how many*, to identify the question. In this example, the question is:

What is the length of a 250. mm metal bar in m?

2. Write the question in mathematical form.

? m = 250. mm

(how many m are there in 250. mm?)

3. What is given?

This is information supplied in the problem. In this example, the given information is:

Length = 250. mm

and the unit we want to find,

New units = m

4. What is known?

Using your knowledge of the SI system, along with applicable tables or notes, try to find relationships between the units in the problem. In this example, both m and mm are units of length in the SI system.

It is also known that the prefix milli- represents 1/1000 or 10^{-3} unit. Thus we have a relationship between m and mm:

$$1 \text{ mm} = 1 \times 10^{-3} \text{ m} \quad \text{OR} \quad 1000 \text{ mm} = 1 \text{ m}$$

Since this relationship will relates mm and m, the two units of concern in this example, it will enable us to solve the problem.

5. Map out the solution.

What steps will be necessary to use the given and known information to solve this problem?. In this example, it will be a simple conversion from mm to m.

$$\text{mm} \rightarrow \text{m}$$

6. Set up conversion factors.

Conversion factors, for a particular problem, are derived from the given and known information. For example, using the relationship

$$1000 \text{ mm} = 1 \text{ m}$$

Two different conversion factors can be written:

$$\frac{1000 \text{ mm}}{1 \text{ m}} \qquad \text{OR} \qquad \frac{1 \text{ m}}{1000 \text{ mm}}$$

Just as the fractions

$$\frac{1}{1} = 1 \qquad \frac{3}{3} = 1 \qquad \frac{568}{568} = 1$$

are all equivalent to "1", so are the conversion factors equal to "1". It is easy to see that fractions such as one over one, three over three, and 568 over 568 are equal to "1", but a fraction such as 1000 mm over 1 m (or 1 m over 1000 mm) may not appear so obvious. Both measurements, 1000 mm and 1 m represent the same length, even if they do not look the same.

7. Set up the solution.

To set up the problem, start with the question in mathematical form and multiply by the appropriate conversion factors. For this problem, the conversion factor can be expressed in two ways. Thus the solution could be written as:

$$? \text{ m} = 250. \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}}$$

OR

$$? \text{ m} = 250. \text{ mm} \times \frac{1000 \text{ mm}}{1 \text{ m}}$$

The choice of conversion factors will be determined by the units we want to eliminate and the final units we want for expressing the answer. For example, if you were to multiple the following fractions:

$$\frac{3}{4} \times \frac{8}{9}$$

You would first cancel between the numerator of one fraction and the denominator of the other fraction:

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}}$$

Then multiply the numbers to get the final answer of 2/3.

When solving problems, both numbers and units will cancel out. Just as in the fractions, above, canceling can only occur between the numerator of one factor and the denominator of another. So, the correct units arrangement of units will be

$$? \text{ m} = 250. \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}}$$

This will allow the units of mm to cancel out leaving the unit of m.

8. Check cancellation of units.

The quickest check of the set-up will be the cancellation of units. If the conversion factors are properly set up and all numbers are correct, unwanted units cancel out leaving the units we want for the final answer.

$$? \text{ m} = 250. \cancel{\text{mm}} \times \frac{1 \text{ m}}{1000 \cancel{\text{mm}}}$$

In this example, the units of mm have canceled out leaving the unit of m.

9. Do the arithmetic.

If you cancel any numbers to make calculations easier, do so, then collect the remaining numbers and do the arithmetic. Arithmetic calculation can be done longhand or with an electronic calculator.

In this example, the 250. can be canceled with the 1000

$$? \text{ m} = \overset{1}{\cancel{250}}. \times \frac{1 \text{ m}}{\underset{4}{\cancel{1000}}}$$

Collecting the numbers, we have:

$$? \text{ m} = \frac{1 \text{ m}}{4}$$

NOTE: It is not necessary to show all the one's.

Solve by dividing 1 by 4 to get the answer:

$= 0.25$ m

Express the final answer using significant figures and including the proper units. In this example, the starting number, 250. mm contained 3 significant figures, so, the final answer must contain 3 significant figures:

$= 0.250$ m

10. Is the answer reasonable?

Take a few seconds to consider the final answer. Does it appear reasonable. In this example, we converted from mm, a small unit, to m, a large unit. There should be less larger units than the original smaller unit. Thus an answer of 0.250 m should appear reasonable, but an answer of 250,000 m should not.

Repeating this process with another example.

Example 2:
A container of salt is reported to have a mass of 73,700 cg. Express this quantity in kg.

1. What is the question?

In this example, the question is:

What is the mass of 73,700 cg salt in kg?

2. Write the question in mathematical form.

? kg = 73,700 cg

(how many kg are there in 73,700 cg?)

3. What is given?

In this example, the given information is the mass of 73,700 cg and the unit we want to find, kg.

4. What is known?

In this example, both cg and kg are units of mass in the SI system.

It is also known that the prefix centi- represents 1/100 or 10^{-2} unit. Thus we have a relationship between cg and g:

$1 \text{ cg} = 1 \times 10^{-2} \text{ g}$ OR $100 \text{ cg} = 1 \text{ g}$

Also, the prefix kilo- represents 1000 units. The relationship between g and kg is:

1 kg = 1000 g

Since these relationships relate cg and kg, the two units of concern in this example, they will enable us to solve the problem.

5. **Map out the solution**.

What steps will be necessary to use the given and known information to solve this problem?. In this example, we will have to convert from cg to g and from g to kg.

$$\text{cg} \rightarrow \text{g} \rightarrow \text{kg}$$

6. **Set up conversion factors**.

Using the relationship 100 cg = 1 g, the conversion factors are:

$$\frac{100\ \text{cg}}{1\ \text{g}} \quad \text{OR} \quad \frac{1\ \text{g}}{100\ \text{cg}}$$

and for the relationship 1000 g = 1 kg, the conversion factors are:

$$\frac{1000\ \text{g}}{1\ \text{kg}} \quad \text{OR} \quad \frac{1\ \text{kg}}{1000\ \text{g}}$$

7. **Set up the solution**.

To set up this problem, the first conversion factor should be written so it eliminates the unit of cg. This step is written as:

$$?\ \text{kg} = 73{,}700\ \text{cg} \times \frac{1\ \text{g}}{100\ \text{cg}}$$

The next step will eliminate the unit of g and leave the unit of kg.

$$?\ \text{kg} = 73{,}700\ \text{cg} \times \frac{1\ \text{g}}{100\ \text{cg}} \times \frac{1\ \text{kg}}{1000\ \text{g}}$$

8. **Check cancellation of units.**

If the conversion factors are properly set up and all numbers are correct, the unwanted units cancel out leaving the units we want for the final answer.

$$?\ \text{kg} = 73{,}700\ \cancel{\text{cg}} \times \frac{1\ \cancel{\text{g}}}{100\ \cancel{\text{cg}}} \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}$$

In this example, the units of cg and g have canceled out leaving the unit of kg.

9. **Do the arithmetic**.

Collect the numbers and do the arithmetic.

$$?\ \text{kg} = 73{,}700 \times \frac{1}{100} \times \frac{1\ \text{kg}}{1000}$$

$$= \frac{73{,}700\ \text{kg}}{100 \times 1000}$$

$= 0.737$ kg

In this example, the starting number, 73,700 cg contained 3 significant figures, so, the final answer must contain 3 significant figures.

10. Is the answer reasonable?

In this example, we converted from cg, a small unit, to kg, a large unit. There should be less larger units than the original smaller unit. Thus an answer of 0.737 kg should appear reasonable.

Example 3:

An automobile is traveling at a speed of 100. km/h on a major highway. Express the speed of the automobile in m/s.

1. What is the question?

In this example, the question is:

What is the speed of 100. km/hr in m/s?

2. Write the question in mathematical form.

$$? \ \frac{\text{m}}{\text{s}} = \frac{100 \text{ km}}{\text{hr}}$$

(how many m/s are there in 100. km/hr?)

3. What is given?

In this example, the given information is the speed of 100 km/hr and the units we want to find, m/s.

4. What is known?

In this example, both km and m are units of length in the SI system and both hr and s are units of time.

It is also known that the prefix kilo- represents 1000 or 10^3 unit. Thus, we have a relationship between km and m:

$$1 \text{ km} = 1000 \text{ m}$$

Looking at the units of time, the relationship between hr and min is:

$$1 \text{ hr} = 60 \text{ min}$$

and the relationship between min and s is:

$$1 \text{ min} = 60 \text{ s}$$

Since these relationships relate km and m and hr and s, the units of concern in this example, they will enable us to solve the problem.

5. Map out the solution.

What steps will be necessary to use the given and known information to solve this problem?. In this example, we will have to convert from km to m and from hr to min to s.

$$\frac{\text{km}}{\text{hr}} \rightarrow \frac{\text{m}}{\text{hr}} \rightarrow \frac{\text{m}}{\text{min}} \rightarrow \frac{\text{m}}{\text{s}}$$

6. Set up conversion factors.

Using the relationship 1000 m = 1 km, the conversion factors are:

$$\frac{1000\text{ m}}{1\text{ km}} \quad \text{OR} \quad \frac{1\text{ km}}{1000\text{ m}}$$

for the relationship 1 hr = 60 min, the conversion factors are:

$$\frac{60\text{ min}}{1\text{ hr}} \quad \text{OR} \quad \frac{1\text{ hr}}{60\text{ min}}$$

and for the relationship 1 min = 60 s, the conversion factors are:

$$\frac{60\text{ s}}{1\text{ min}} \quad \text{OR} \quad \frac{1\text{ min}}{60\text{ s}}$$

7. Set up the solution.

To set up this problem, the first conversion factor should be written so it eliminates either the unit of km or the unit of hr. For this example, the unit of km will be eliminated using the conversion factor of 1000 m/1 km. This step is written as:

$$?\ \frac{\text{m}}{\text{s}} = \frac{100\text{ km}}{\text{hr}} \times \frac{1000\text{ m}}{1\text{ km}}$$

The next step will eliminate the unit of hr using the conversion factor 1 hr/60 min:

$$?\ \frac{\text{m}}{\text{s}} = \frac{100\text{ km}}{\text{hr}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ hr}}{60\text{ min}}$$

The next step will eliminate the unit of min using the conversion factor 1 min/60 s:

$$?\ \frac{\text{m}}{\text{s}} = \frac{100\text{ km}}{\text{hr}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ hr}}{60\text{ min}} \times \frac{1\text{ min}}{60\text{ s}}$$

8. Check cancellation of units.

If the conversion factors are properly set up and all numbers are correct, the unwanted units cancel out leaving the units we want for the final answer.

$$?\ \frac{\text{m}}{\text{s}} = \frac{100\ \cancel{\text{km}}}{\cancel{\text{hr}}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{hr}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\text{ s}}$$

In this example, the units of km/hr have canceled out leaving the unit of m/s.

9. Do the arithmetic.

Collect the numbers and do the arithmetic.

$$? \frac{m}{s} = \frac{100}{1} \text{ x } \frac{1000 \text{ m}}{1} \text{ x } \frac{1}{60} \text{ x } \frac{1}{60 \text{ s}}$$

$$= \frac{100 \text{ x } 1000 \text{ m}}{60 \text{ x } 60 \text{ s}}$$

$$= \frac{100{,}000 \text{ m}}{3600 \text{ s}}$$

$$= 27.777777 \text{ m/s}$$

In this example, the starting number, 100. km/hr contained 3 significant figures, so, the final answer must contain 3 significant figures.

$$= 27.8 \text{ m/s}$$

10. Is the answer reasonable?

In this example, it is more difficult to judge the final answer since two units were changed. We converted from km, a large unit to m, a smaller unit, but at the same time the time factor was also reduced from 1 hr to 1 s. The final answer of 27.8 m/s appears to be reasonable.

Problems: Dimensional Analysis

1. Convert 3.56 g to cg.
2. Convert 42.5 mL to dL.
3. Convert 204 dm to km.
4. Convert 45.4 kg to Mg.
5. Convert 2.60 cL to µL.
6. Convert 254 pm to nm.
7. The mass of a hydrogen atom is 1.67 x 10^{-24} g. Express this mass in pg.
8. The diameter of a carbon atom is 182 pm. What is the diameter in a) nm b) cm c) m ?
9. In a gas chromatography experiment, a 5.0 µL sample of gasoline was analyzed. Express this volume in mL.
10. The speed of light is 3.0 x 10^{10} cm/s. Calculate the length of a light year in a) m b) km. (A light year is the distance light travels in one year [365 days])
11. How many dg are there in 1.0 kg of flour?
12. A jogger runs a km in 6.0 minutes. Calculate her speed in a) km/hr b) m/min c) cm/s.